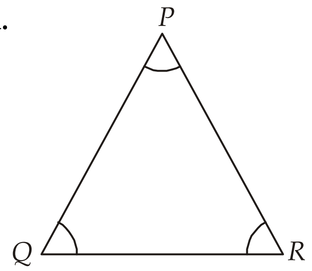
Chapter 14 : Congruence of Triangles

ANSWER KEYS

EXERCISE 14.1

1.



Six elements of ΔPQR are its three sides and three angles.

Three sides: PQ, QR and PR.

Three angles: $\angle P$, $\angle Q$ and $\angle R$.

2. $\because$ $\Delta XYZ \cong \Delta RPQ$ under the correspondence $XYZ \leftrightarrow RPQ$.

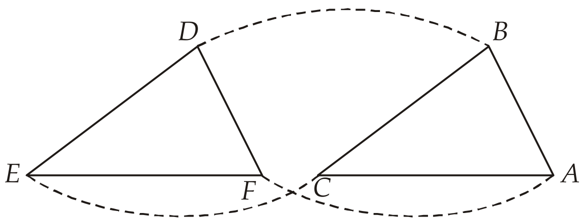
Therefore, all corresponding congruent parts of the triangles are:

$\angle X \leftrightarrow \angle R$, $\angle Y \leftrightarrow \angle P$, $\angle Z \leftrightarrow \angle Q$ and side $XY \leftrightarrow$ side RP, side $YZ \leftrightarrow$ side PQ, side $ZX \leftrightarrow$ side QR.

3. $\Delta DEF \cong \Delta BCA$, under the correspondence $DEF \leftrightarrow BCA$.

This means $D \leftrightarrow B$; $E \leftrightarrow C$; $F \leftrightarrow A$.

Therefore, the corresponding parts of ΔBCA are :



(*i*) $\angle F \leftrightarrow \angle A$

(*ii*) $DE \leftrightarrow BC$

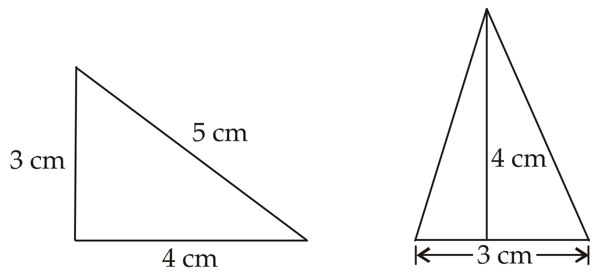
(*iii*) $\angle D \leftrightarrow \angle B$

(*iv*) $EF \leftrightarrow CA$

(*v*) $DF \leftrightarrow BA$

4. (*i*) No, the triangles equal in area may not be congruent.

Consider two triangles as shown in the figures given below:



These triangles are equal in area but they are not congruent.

(*ii*) Yes, congruent rectangles have equal area.

Two rectangles are congruent, if their lengths and breadths are same *i.e.*, same area.

(*iii*) Yes, the squares having equal area are congruent.

$\therefore$ Two squares are congruent if they have same side length *i.e.*, same area.

(*iv*) No, all squares are not congruent as they do not have same side length.

(*v*) No, circles with same centre are not congruent as they have different radii.

$\because$ Circles with same radii are congruent.

5. Given that :

$\angle BOD \cong \angle AOC$

To prove :

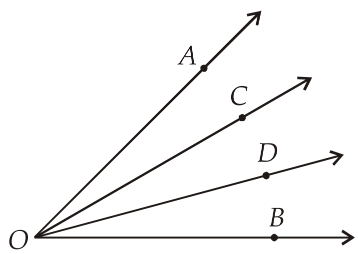
$\angle BOC \cong \angle AOD$

Proof :

$\because$ $\angle BOD \cong \angle AOC$

If two angles are congruent, their measures are same.

$\therefore$ $\angle BOD = \angle AOC$



Adding $\angle COD$ on both sides, we get

$$\angle BOD + \angle COD = \angle AOC + \angle COD$$

$\Rightarrow \quad \angle BOC = \angle AOD$

If two angles have the same measure, they are congruent.

$\therefore \quad \angle BOC \cong \angle AOD$. (Hence proved)

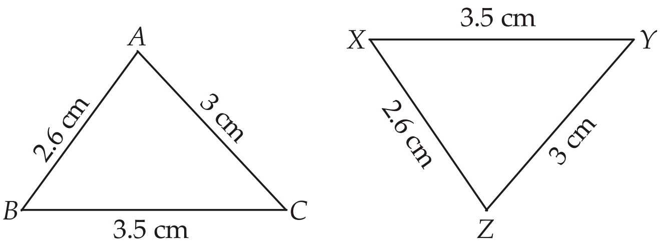
EXERCISE 14.2

1. In ΔABC and ΔZXY,

$AB = ZX = 2.6$ cm (given)

$BC = XY = 3.5$ cm (given)

$CA = YZ = 3$ cm (given)



Therefore, ΔABC and ΔZXY are congruent.

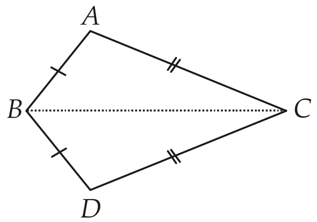
$\Delta ABC \cong \Delta ZXY$ (By SSS congruence rule)

2. (*i*) In ΔBAC and ΔBDC,

$BA = BD$ (given)

$AC = DC$ (given)

BC is common.

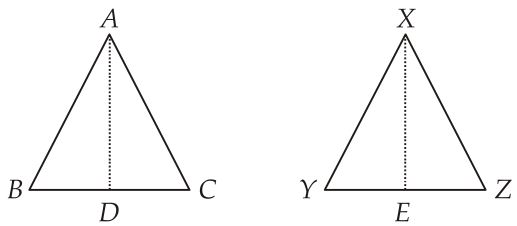


Hence, $\Delta BAC \cong \Delta BDC$ (By SSS congruence rule)

(*ii*) $\angle ABC = \angle DBC$ (by C.P.C.T.)

Hence, $\angle ABC = \angle CBD$ ($\because \angle CBD = \angle DBC$)

3.



Given that : In ΔABC and ΔXYZ, $AB = XY$, $BC = YZ$ and median AD = median XE.

To Prove : $\Delta ABD \cong \Delta XYE$

Proof : $\because \quad BC = YZ$ (given)

$\therefore \quad 2BD = 2YE$ ($\because$ AD and XE are median)

$BD = YE$ ... (*i*)

In ΔABD and ΔXYE,

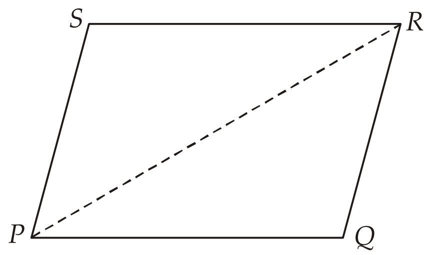
$AB = XY$ (given)

Median AD = Median XE

$BD = YE$ [from (*i*)]

Thus, $\Delta ABD \cong \Delta XYE$ (by SSS congruence rule)

4. In the parallelogram $PQRS$, we have $PQ = RS$ and $QR = SP$. PR is diagonal.



Now, in ΔPQR and ΔRSP,

$PQ = RS$ (opposite sides of a parallelogram)

$QR = SP$ (opposite sides of a parallelogram)

Diagonal PR is common.

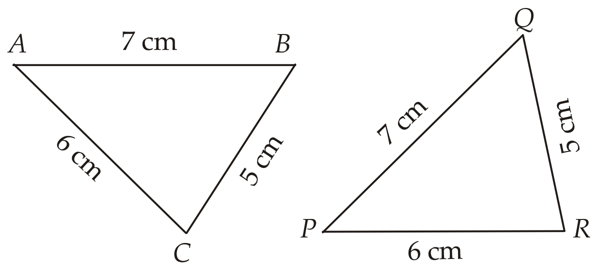
Hence, $\Delta PQR \cong \Delta RSP$ (by SSS congruence rule)

5. (*i*) In ΔABC and ΔPQR,

$AB = PQ = 7$ cm,

$BC = QR = 5$ cm,

$CA = RP = 6$ cm



$\therefore \quad \angle A \leftrightarrow \angle P, \angle B \leftrightarrow \angle Q, \angle C \leftrightarrow \angle R$

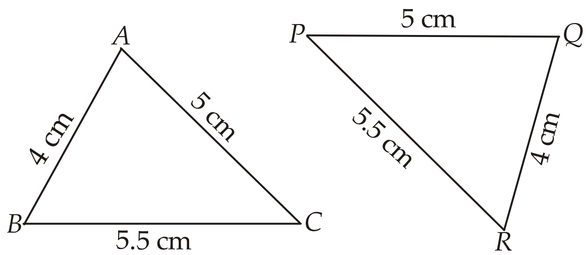
$\therefore \quad \Delta ABC \cong \Delta PQR$ (By SSS congruence rule)

(*ii*) We have,

$AB = QR = 4$ cm,

$BC = RP = 5.5$ cm

$CA = PQ = 5$ cm



$\therefore \quad \angle A \leftrightarrow \angle Q, \angle B \leftrightarrow \angle R, \angle C \leftrightarrow \angle P$

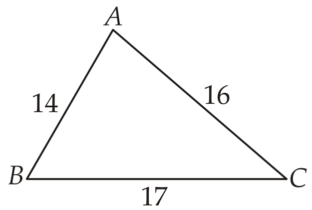
Hence, $\Delta ABC \cong \Delta QRP$ (By SSS congruence rule)

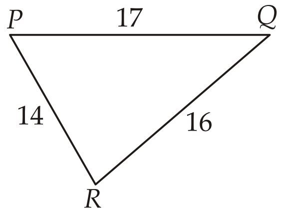
6. (*i*) In ΔABC and ΔRPQ

$AB = RP = 14$ cm

$BC = PQ = 17$ cm

$CA = QR = 16$ cm





Thus, $\Delta ABC \cong \Delta RPQ$ (By SSS congruence rule)

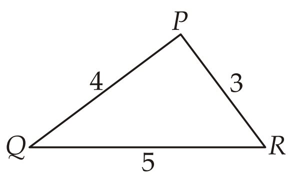
$\therefore \quad \angle A = \angle R, \angle B = \angle P, \angle C = \angle Q.$

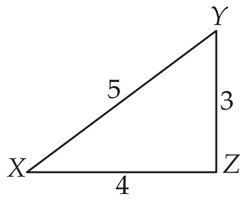
(*ii*) In ΔPQR and ΔZXY,

$PQ = ZX = 4$ cm

$QR = XY = 5$ cm

$RP = YZ = 3$ cm





$\therefore \quad \Delta PQR \cong \Delta ZXY$ (By SSS congruence rule)

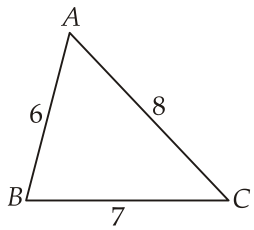
$\therefore \quad \angle P \leftrightarrow \angle Z, \angle Q \leftrightarrow \angle X, \angle R \leftrightarrow \angle Y.$

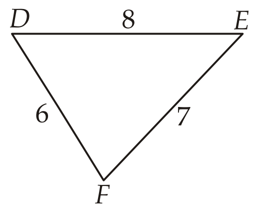
(*iii*) In ΔABC and ΔDFE,

$AB = DF = 6$ cm,

$BC = FE = 7$ cm,

$CA = ED = 8$ cm





$\therefore \quad \Delta ABC \cong \Delta DFE$ (By SSS congruence rule)

$\angle A \leftrightarrow \angle D, \angle B \leftrightarrow \angle F, \angle C \leftrightarrow \angle E$

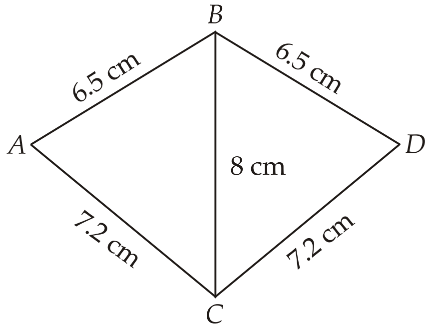
(Corresponding angles of congruent triangles)

7. In ΔABC and ΔDBC,

$AB = DB = 6.5$ cm

BC is common.

$CA = CD = 7.2$ cm



Thus, $\Delta ABC \cong \Delta DBC$(By SSS congruence rule)

$\therefore \quad \angle A = \angle D, \angle ABC = \angle DBC, \angle ACB = \angle DCB$

(Corresponding angles of congruent triangles)

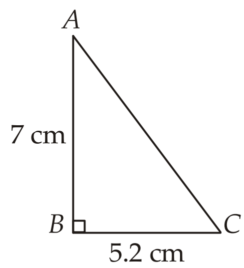
EXERCISE 14.3

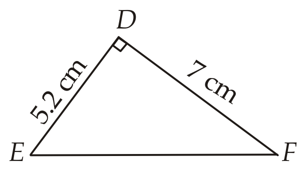
1. (*i*) In ΔABC and ΔFDE,

$AB = FD = 7$ cm

$\angle ABC = \angle FDE = 90°$

$BC = DE = 5.2$ cm





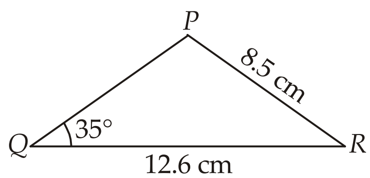
$\therefore \quad \Delta ABC \cong \Delta FDE$ (By SAS congruence rule)

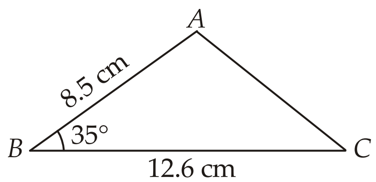
(*ii*) In ΔPRQ and ΔABC,

$PR = AB = 8.5$ cm

$RQ = BC = 12.6$ cm

But included $\angle R \neq$ included $\angle B$





So we cannot say that the triangles are congruent.

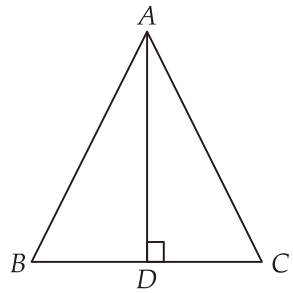
2. **Given that :** In ΔABC, altitude AD bisects BC.

To prove : $\Delta ADB \cong \Delta ADC$

Proof : Since, altitude AD bisects BC.

$\therefore \quad BD = DC \quad \dots (i)$

Now, in ΔADB and ΔADC, AD is common.



$\angle ADB = \angle ADC = 90°$ ($\because$ AD is an altitude)

$DB = DC$ [From (i)]

$\therefore \quad \Delta ADB \cong \Delta ADC$ (By SSS congruence rule)

$\therefore \quad AB = AC$

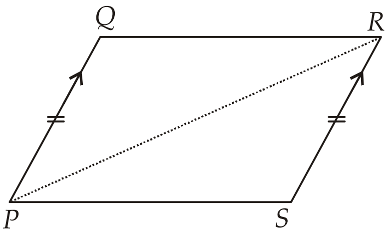
(Corresponding sides of congruent triangles)

$\therefore$ Equal pairs of sides of these two triangles are $AB = AC$, $DB = DC$ and AD common.

3. **Given that :** $PQ = SR$ and $PQ \parallel SR$.

To prove : $\Delta PSR \cong \Delta RQP$

Construction : Draw a diagonal PR.



Proof : $\because$ $PQ \parallel SR$

$\therefore \quad \angle QPR = \angle SRP \quad \dots (i)$ (Alternate angles)

Now, in ΔPSR and ΔRQP,

$SR = QP$ (given)

$\angle SRP = \angle QPR$ [from (i)]

RP is common.

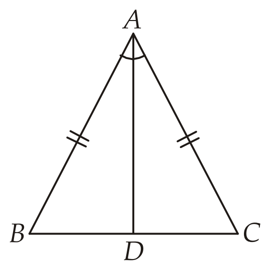
$\therefore \quad \Delta PSR \cong \Delta RQP$ (By SAS congruence rule)

Hence, $PS = QR$

(Corresponding sides of congruent triangles)

4. **Given that :** In ΔABC, AD is the bisector of $\angle A$ and $AB = AC$.

To prove : $\angle B = \angle C$



Proof : Since, AD bisects $\angle A$

$\therefore \quad \angle BAD = \angle CAD \quad \dots (i)$

In ΔABD and ΔACD,

$AB = AC$ (given)

$\angle BAD = \angle CAD$ [from (i)]

AD is common.

$\therefore \quad \Delta ABD \cong \Delta ACD$ (By SAS congruence rule)

Hence, $\angle B = \angle C$

(Corresponding angles of congruent triangles)

Angles opposite to equal sides are equal.

5. **Given that :** $ABCD$ is a quadrilateral and AC is a diagonal.

To prove : $\Delta ABC \cong \Delta CDA$

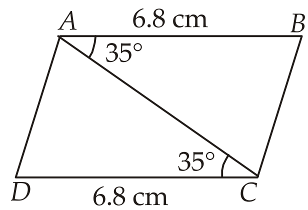
Proof : Since, diagonal AC divides the quadrilateral $ABCD$ in two triangles.

In ΔABC and ΔCDA,

$BA = DC = 6.8$ cm (given)

$\angle BAC = \angle DCA = 35°$ (given)

AC is common.



$\therefore \quad \Delta ABC \cong \Delta CDA$ (By SAS congruence rule)

$\because \quad \angle BAC = \angle DCA = 35°$ (Alternate angles)

AC is transversal.

$\therefore \quad AB \parallel CD$.

EXERCISE 14.4

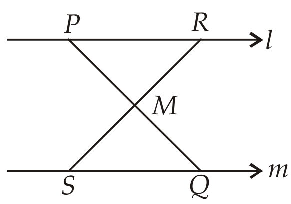
1. **Given that :** Line $l \parallel m$, and M is the mid point of line segment PQ.

To prove : M is also mid point of RS.

Proof : $\because$ $l \parallel m$

$\therefore \quad \angle MPR = \angle MQS$ and $\angle MRP = \angle MSQ \quad \dots (i)$

(Alternate angles)



$\therefore$ M is mid point of PQ.

$\therefore \quad MP = MQ \quad \dots (ii)$

Now, in ΔMPR and ΔMQS,

$\angle MPR = \angle MQS$ [from (i)]

Side $MP = MQ$ [from (*ii*)]

$\angle MRP = \angle MSQ$ [from (*i*)]

$\therefore$ $\Delta MPR \cong \Delta MQS$ (By ASA congruence rule)

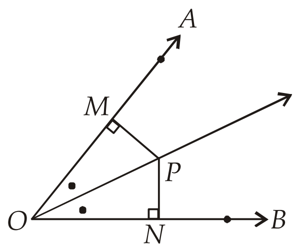
$\therefore$ $MS = MR$ (By C.P.C.T.)

Hence, M is also the mid point of RS.

2. **Given that :** OP is bisector of $\angle AOB$, $PM \perp OA$ and $PN \perp OB$.

To prove : $PM = PN$

Proof : Since, OP is the bisector of $\angle AOB$.



$\therefore$ $\angle AOP = \angle POB$

or $\angle MOP = \angle NOP$ ... (*i*)

In ΔPMO and ΔPNO,

$\angle MOP = \angle NOP$ [from (*i*)]

OP is common.

$\angle PMO = \angle PNO$ (given)

$\therefore$ $\Delta PMO \cong \Delta PNO$ (By ASA congruence rule)

$\therefore$ $PM = PN$

(Corresponding sides of congruent triangles)

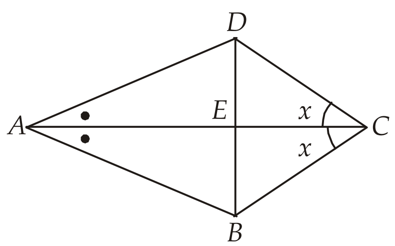
Hence proved.

3. **Given that :** In quadrilateral $ABCD$, diagonal AC bisects $\angle BAD$ and $\angle BCD$.

To prove : AC bisects BD at right angles.

Proof : In ΔACD and ΔACB,

$\angle DAC = \angle BAC$ ($\because$ AC bisects $\angle BAD$)



AC is common.

$\angle ACD = \angle ACB$ ($\because$ AC bisects $\angle BCD$)

$\therefore$ $\Delta ACD \cong \Delta ACB$ (By ASA congruence rule)

$\therefore$ $CD = CB$ (By C.P.C.T.) ...(*i*)

Now, in ΔCED and ΔCEB,

$CD = CB$ [from (*i*)]

$\angle DCE = \angle BCE = x$ (given)

CE is common.

$\therefore$ $\Delta CED \cong \Delta CEB$ (by SAS congrunce rule)

$\therefore$ $DE = BE$

(Corresponding sides of congruent triangles)

and $\angle CED = \angle CEB$ (*ii*)

(Corresponding angles of congruent triangles)

$\because$ $\angle CED + \angle CEB = 180°$

(Linear pair of angles)

$\therefore$ $\angle CED + \angle CED = 180°$

[$\because$ $\angle CED = \angle CEB$ from (*ii*)]

$\therefore$ $2\angle CED = 180°$

$$\angle CED = \frac{180°}{2} = 90°$$

$\therefore$ $\angle CED = \angle CEB = 90°$

Hence, AC bisects BD at right angles.

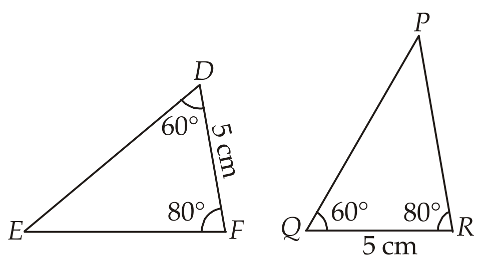
4. (*i*) In ΔDEF and ΔQPR,

$\angle D = \angle Q = 60°$

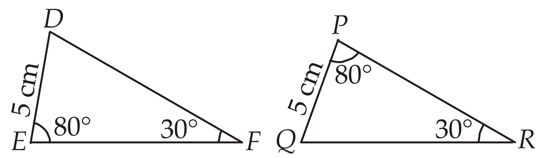
$DF = QR = 5$ cm

$\angle F = \angle R = 80°$

$\therefore$ $\Delta DEF \cong \Delta QPR$ (By ASA congruence rule)



(*ii*)



In ΔDEF,

$\angle D = 180° - (\angle E + \angle F)$

$= 180° - (80° + 30°)$

$= 180° - 110°$

$= 70°$

In ΔPQR,

$\angle Q = 180° - (\angle P + \angle R)$

$= 180° - (80° + 30°)$

$= 180° - 110°$

$= 70°$

In ΔDEF and ΔQPR,

$\angle D = \angle Q = 70°$

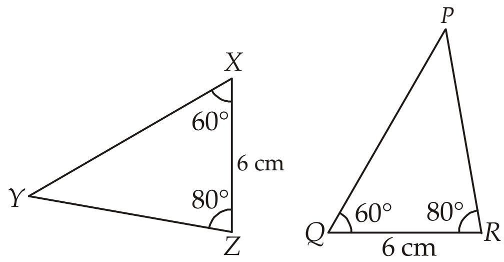
$DE = QP = 5$ cm

$\because$ $\angle E = \angle P = 80°$

Thus,

$\therefore$ $\Delta DEF \cong \Delta QPR$ (By ASA congruence rule)

(*iii*)



In ΔYXZ and ΔPQR,

$\because \quad \angle X = \angle Q = 60°$

side XZ = side QR

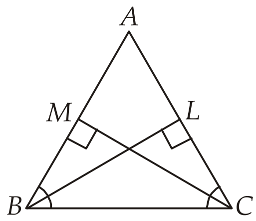
$\angle Z = \angle R = 80°$

$\therefore \quad \Delta YXZ \cong \Delta PQR$ (By ASA congruence rule)

5. **Given that :** In ΔABC, $\angle B = \angle C$, BL and CM bisect $\angle B$ and $\angle C$ respectively.

To prove : $BL = CM$

Proof : $\because$ $\angle B = \angle C$



$2\angle LBC = 2\angle MCB$

(BL and CM bisect $\angle B$ and $\angle C$)

$\angle LBC = \angle MCB$ .. (*ii*)

ΔBMC and ΔCLB,

$\angle CBM = \angle BCL$ (given)

BC is common.

$\angle MCB = \angle LBC$ [from (*i*)]

$\therefore \quad \Delta BMC \cong \Delta CLB$ (By ASA congruence rule)

$\therefore \quad BL = CM$

(Corresponding sides of congruent triangles)

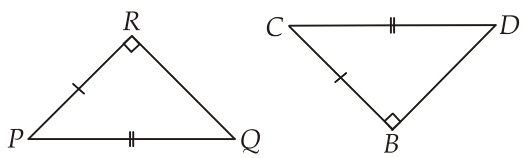
EXERCISE 14.5

1. (*i*) In ΔPRQ and ΔCBD,

$\angle PRQ = \angle CBD = 90°$ (given)

hypotenuse QP = hypotenuse DC. (given)

$PR = CB$ (given)



Hence, $\Delta PRQ \cong \Delta CBD$

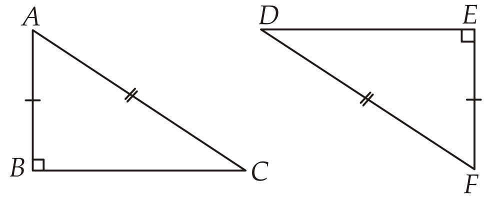
(By RHS congruence rule)

(*ii*) In ΔABC and ΔFED,

$\angle ABC = \angle FED = 90°$

hypotenuse CA = hypotenuse DF

side AB = side FE



$\therefore \quad \Delta ABC \cong \Delta FED$ (By RHS congruence rule)

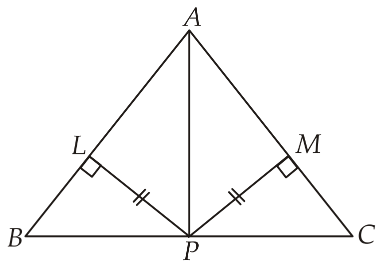
2. **Given that :** In ΔABC, P is a point on side BC such that $PL \perp AB$ and $PM \perp AC$.

To prove : AP bisects $\angle BAC$.

Proof : In ΔALP and ΔAMP,

$\angle ALP = \angle AMP = 90°$

($\because$ $PL \perp AB$ and $PM \perp AC$)



Hypotenuse AP is common.

side LP = side MP

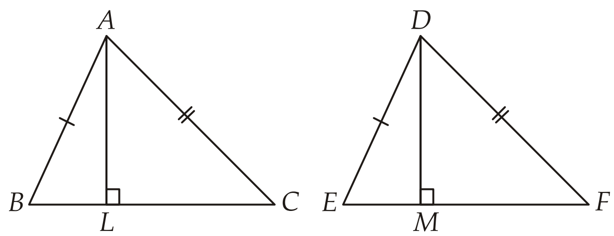
$\therefore \quad \Delta ALP \cong \Delta AMP$ (By RHS congruence rule)

$\therefore \quad \angle PAL = \angle PAM$

(Corresponding angles of congruent triangles)

Hence, AP bisects $\angle BAC$. Proved

3.



Given that : Two triangles ABC and DEF are such that

$AL \perp BC$, $DM \perp EF$

$AB = DE$, $AC = DF$, $AL = DM$

To prove : $\Delta ABC \cong \Delta DEF$

Proof : In ΔALB and ΔDME,

$\angle ALB = \angle DME = 90°$

($\because$ AL and DM are altitudes)

$AL = DM$ (given)

hypotenuse AB = hypotenuse DE (given)

So, $\Delta ABL \cong \Delta DME$ (By RHS congruence rule)

$\Rightarrow \angle B = \angle E$ (CPCT)

$\angle BAL = \angle EDM$ (CPCT)

In ΔALC and ΔDMF

$\angle ALC = \angle DMF = 90°$

($\because$ AL and DM are altitudes)

$AL = DM$ (given)

hypotenuse AC = hypotenuse DF (given)

$\therefore \Delta ALC \cong \Delta DMF$ (By RHS congruence rule)

$\therefore \angle C = \angle F$ (By CPCT)

$\angle CAL = \angle FDM$ ...(*ii*) (By CPCT)

Now,

$\angle BAL = \angle EDM$ [from (*i*)]

Adding $\angle CAL$ on both sides, we get

$\Rightarrow \angle BAL + \angle CAL = \angle EDM + \angle CAL$

$\Rightarrow \angle BAL + \angle CAL = \angle EDM + \angle FDM$ [from (*ii*)]

$\Rightarrow \angle A = \angle D$ ...(*iii*)

Now, in ΔABC and ΔDEF

$AB = DE$ (given)

$\angle A = \angle D$ [from (*iii*)]

$AC = DF$ (given)

Hence, $\Delta ABC \cong \Delta DEF$. (By SAS rule)

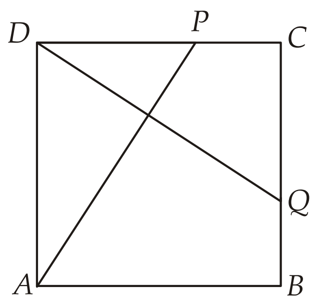
4. **Given that :** $ABCD$ is a square. P and Q are points on DC and BC respectively, such that $AP = DQ$

To prove : $\Delta ADP \cong \Delta DCQ$

Proof : $\because$ $ABCD$ is a square.

$\therefore \angle A = \angle B = \angle C = \angle D = 90°$.

and $AB = BC = CD = DA$ ... (*i*)



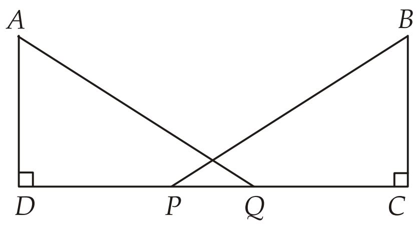
In ΔADP and ΔDCQ,

$\angle ADP = \angle DCQ = 90°$

hypotenuse AP = hypotenuse DQ (given)

side $AD = DC$ [from (*i*)]

$\therefore \Delta ADP \cong \Delta DCQ$ (By RHS congruence rule)

5. 

Given that : $AD \perp CD$, $BC \perp CD$ and $AQ = BP$, $DP = CQ$.

To prove : $\angle DAQ = \angle CBP$

Proof : $\because$ $DP = CQ$

Adding PQ on both sides, we get

$DP + PQ = CQ + PQ$

$DQ = CP$ ... (*i*)

ΔDAQ and ΔCBP,

$\angle ADQ = \angle BCP = 90°$ (given)

hypotenuse AQ = hypotenous PB (given)

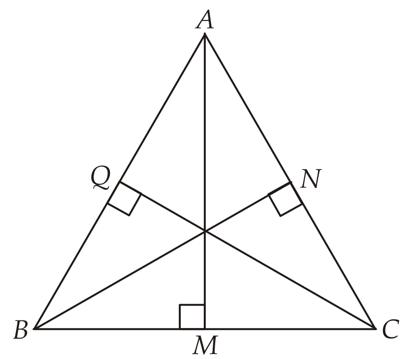
$DQ = CP$ [from (*i*)]

$\therefore \Delta ADQ \cong \Delta BCP$ (By RHS congruence rule)

$\therefore \angle DAQ = \angle CBP$

(Corresponding angle of congruent triangles)

6. **Given that :** In ΔABC, $AM \perp BC$, $BN \perp AC$ and $CQ \perp AB$ and $AM = BN = CQ$



To prove : ΔABC is an equilateral triangle.

Proof : In right triangles BNC and CQB,

Hypotenuse BC is common.

$BN = CQ$

$\angle BNC = \angle CQB = 90°$

($\because$ $BN \perp AC$, $CQ \perp AB$)

So, $\Delta BNC \cong \Delta CQB$ (By RHS congruence rule)

$\Rightarrow \angle B = \angle C$ (CPCT)

$\Rightarrow AC = AB$ ...(*i*) (Sides opposite to equal angles are equal)

Similarly, $\Delta ABN \cong \Delta ABM$

$\Rightarrow \angle B = \angle A$ (CPCT)

$\Rightarrow AC = BC$ ...(*ii*) (Sides opposite to equal angles are equal)

From (*i*) and (*ii*), we get

$AB = BC = CA$

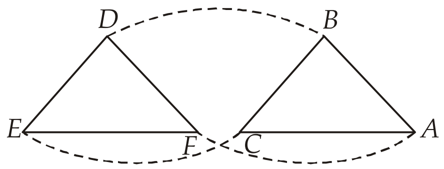
Hence, ΔABC is an equilateral triangle.

MULTIPLE CHOICE QUESTIONS

1. The bisector of the vertical angle of an isosceles triangle bisects the base at 90°.

Hence, option (*b*) is correct.

2. $\because \Delta DEF \cong \Delta BCA,$

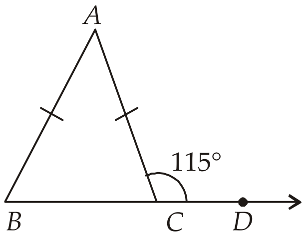


$\therefore \quad D \leftrightarrow B,\ E \leftrightarrow C,\ F \leftrightarrow A$

$\therefore \quad \overline{EF} \leftrightarrow \overline{CA}$

Hence, option (*c*) is correct.

3.



$\because \quad \angle ACB + \angle ACD = 180°$ (Linear pair of angles)

$\Rightarrow \quad \angle ACB = 180° - 115°$

$\Rightarrow \quad \angle ACB = 65°$

$\because \quad AB = AC$

$\therefore \quad \angle ACB = \angle CBA = 65°$

Now, in ΔABC,

$\angle BAC + \angle ACB + \angle CBA = 180°$

$\Rightarrow \quad \angle BAC + 65° + 65° = 180°$

$\Rightarrow \quad \angle BAC + 130° = 180°$

$\Rightarrow \quad \angle BAC = 180° - 130°$

$\Rightarrow \quad \angle BAC = 50°$

Hence, option (*d*) is correct.

4. In ΔABC

$\because \quad AB = AC$

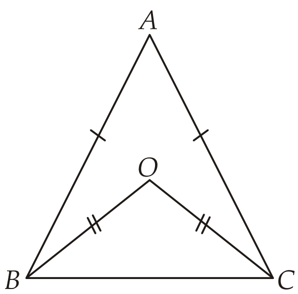
$\therefore \quad \angle ACB = \angle ABC \qquad ...(i)$

and, in ΔOBC,

$\because \quad OB = OC$

$\therefore \quad \angle OCB = \angle OBC \qquad ...(ii)$

(Angle opposite to equal sides are equal)



Now, we have

$\angle ABC = \angle ACB$

$\Rightarrow \quad \angle ABO + \angle OBC = \angle OCA + \angle OCB$

$\Rightarrow \quad \angle ABO + \angle OBC = \angle OCA + \angle OBC \qquad$ [from (*ii*)]

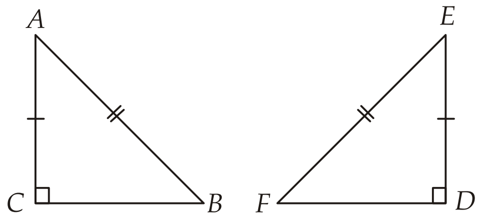
$\Rightarrow \quad \angle ABO = \angle OCA$

$\Rightarrow \quad \dfrac{\angle ABO}{\angle OCA} = 1$

Hence, option (*a*) is correct.

5. In right triangles ABC and DEF, triangles hypotenuse AB = hypotenuse EF and side $AC = DE$.

By RHS congruence rule, these triangle are congruent.

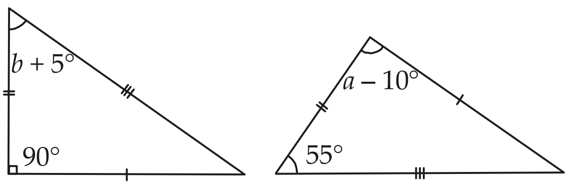


$A \leftrightarrow E,\ B \leftrightarrow F,\ C \leftrightarrow D$

Hence, $\Delta ABC \cong \Delta EFD$.

Hence, option (*d*) correct.

6. $\because$ Both triangles are congruent.



$\therefore \quad b + 5° = 55°$

$\Rightarrow \quad b = 55° - 5° = 50°$

and $\quad 90° = a - 10°$ (Corresponding parts)

or $\quad a - 10° = 90°$

$\Rightarrow \quad a = 90° + 10° = 100°$

Thus, $\quad a = 100°$ and $b = 50°$

Hence, option (*b*) is correct.

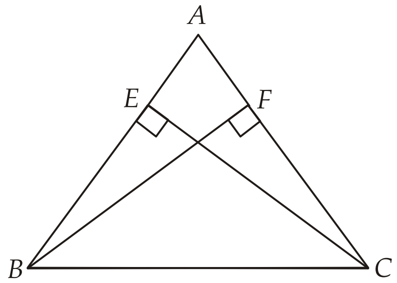
MENTAL MATHS CORNER

A. Fill in the blanks:

1. Two line segments are congruent, if they have **same length**.
2. Two squares are congruent, if they have **same side length**.
3. Two angles are congruent, if they have **same measure**.
4. Two circles are congruent, if they have **same radii**.
5. Two rectangles are congruent, if they have **same length and same breadth**.
6. Two triangles are congruent, if they have **all parts equal**.

7. The figures having the same area **need not be** congruent.
8. Among two congruent angles, one has a measure of 55°, the measure of other angle is **55°**.
9. If altitude CE and BF of a ΔABC are equal, then AB = ***AC***.

Since, in ΔBFC and ΔCEB,



$\angle BFC = \angle CEB = 90°$ ($BF \perp AC$ and $CE \perp AB$)

BC is common.

$BF = CE$

$\therefore \quad \Delta BFC \cong \Delta CEB$ (By RHS congruence rule)

$\therefore \quad \angle CBE = \angle BCF$

(Corresponding angles of congruent triangles)

or $\quad \angle CBA = \angle BCA$

$\therefore \quad AB = AC$.

10. In a ΔABC, if $\angle A = \angle C$, then AB = ***BC***.

Sides opposite to equal angles are equal.

B. True or False:

1. If two figures have same area, then they are congruent. **(False)**
2. Two circles with equal radii are congruent. **(True)**
3. If three angles of one triangle are equal to three corresponding angles of another triangle, then two triangles are congruent. **(False)**

$\because$ One of them may be enlarged copy of the other.

So, the two triangles need not be congruent.

4. Two squares are always congruent. **(False)**
5. If three sides of a triangle are equal to corresponding three sides of the other triangle, then the triangles are congruent. **(True)**
6. If two triangles are congruent, then six elements of one triangle are equal to corresponding six elements of the other triangle. **(True)**
7. If two sides and one angle of a triangle are equal to two sides and one angle of the other triangle, then the triangles are congruent. **(False)**

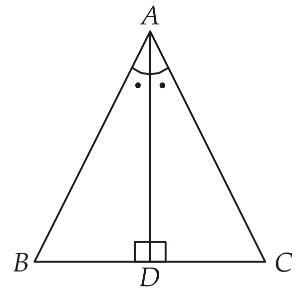
$\because$ The angle may not be the included angle of two sides.

So, the triangles are not congruent.

8. If two triangles are equal in area, they are congruent. **(False)**
9. If the hypotenuse of right triangle is equal to the hypotenuse of another right triangle, then the triangles are congruent. **(False)**

$\because$ Two right triangles are congruent, if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.

10. In the given figure, AD bisects $\angle A$ and $AD \perp BC$, then



(*i*) In ΔADB and ΔADC,

$\angle ADB = \angle ADC = 90°$ (given)

AD is common.

$\angle BAD = \angle CAD$ ($\because$ AD bisects $\angle BAC$)

Hence,

$\Delta ADB \cong \Delta ADC$ (By *ASA* congruence rule) **(True)**

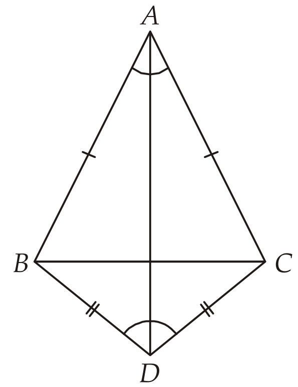
(*ii*) $BD = DC$ **(True)**

(Corresponding sides of congruent triangles)

REVIEW EXERCISE

1. **Given that :** ABC is an isosceles triangle such that $AB = AC$ and $BD = CD$.

To prove : AD bisects $\angle A$ and $\angle D$.



Proof : In ΔABD and ΔACD,

$AB = AC$ (given)

$BD = CD$ (given)

AD is common.

$\therefore \quad \Delta ABD \cong \Delta ACD$ (By SSS congruence rule)

$\therefore \quad \angle DAB = \angle DAC$

(Corresponding angles of congruent triangles)

and $\quad \angle BDA = \angle CDA$

(Corresponding angles of congruent triangles)

Hence, AD bisects $\angle BAC$ and $\angle BDC$ (Proved)

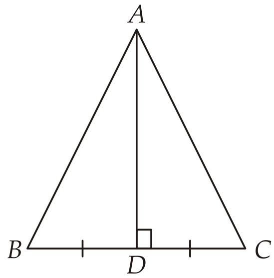
2. **Given that :** In ΔABC, altitude AD bisects BD, such that

$BD = CD$

$AD \perp BC$

To prove : $\Delta ABD \cong \Delta ADC$

Proof : In ΔADB and ΔADC,



$BD = CD$ (given)

$\angle BDA = \angle CDA$ ($\because$ $AD \perp BC$)

AD is common.

$\therefore$ $\Delta ADB \cong \Delta ADC$ (By SAS congruence rule)

$\therefore$ $AB = AC$

(By corresponding sides of congruent triangles)

$BD = CD$

(By corresponding angles of congruent triangles)

$\angle DBA = \angle DCA$

(By corresponding angles of congruent triangles)

$\angle BAD = \angle CAD$

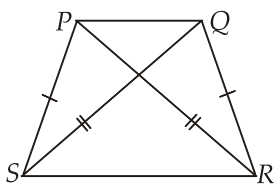
(Corresponding angles of congruent triangles)

3. **Given that :** In quadrilateral $PQRS$,

$PS = QR$, $PR = SQ$.

To prove : $\angle PSQ = \angle QRP$ and

$\angle SPQ = \angle RQP$



Proof : In ΔPSQ and ΔQRP,

$PS = QR$ (given)

$SQ = RP$ (given)

Side PQ is common.

$\therefore$ $\Delta PSQ \cong \Delta QRP$ (By SSS congruence rule)

$\therefore$ $\angle PSQ = \angle QRP$

and $\angle SPQ = \angle RQP$

(Corresponding angles of congruent triangles)

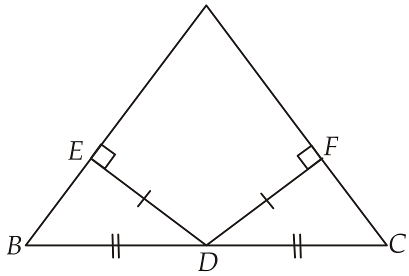
and $\angle SPQ = \angle RQP$

(Corresponding angles of congruent triangles)

4. **Given that :** In ΔABC, $DE = DF$

$BD = DC$ and $DE \perp AB$ and $DF \perp AC$.

To prove : $AB = AC$.



Proof : In ΔBED and ΔCFD,

$\angle BED = \angle CFD$ ($\because$ $DE \perp AB$ and $DF \perp AC$)

hypotenuse DB = hypotenuse DC (given)

side DE = side DF

Thus,

$\Delta BED \cong \Delta CFD$ (By RHS congruence rule)

$\therefore$ $\angle B = \angle C$

(Corresponding angles of congruent triangles)

In ΔABC,

$\angle C = \angle B$

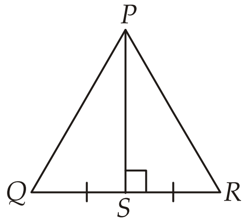
$\Rightarrow$ $AB = AC$ (Proved)

5. (*i*) In ΔPSQ and ΔPSR, $QS = RS$ (given)

$\angle PSQ = \angle PSR = 90°$ (given)

PS is common.

$\therefore$ $\Delta PSQ \cong \Delta PSR$ (By SAS congruence rule)



(*ii*) In ΔABC and ΔADC,

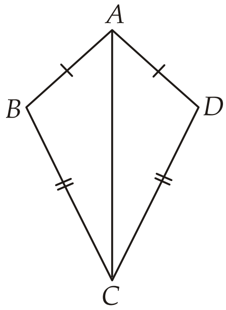
$AB = AD$

$BC = DC$

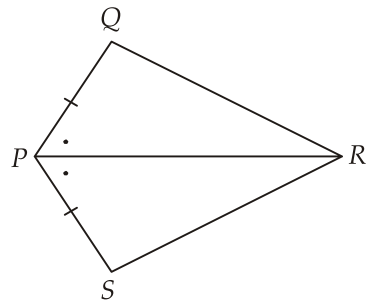
AC is common.

$\therefore$ $\Delta ABC \cong \Delta ADC$

(By SSS congruence rule)



(iii) In ΔPQR and ΔPSR,



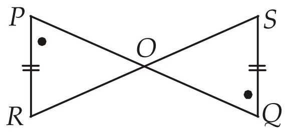
$PQ = PS$

$\angle QPR = \angle SPR$

PR is common.

$\therefore \quad \Delta PQR \cong \Delta PSR$ (By SAS congruence rule)

(iv)



Let PQ and RS intersect at point O.

$\angle POR = \angle QOS$ (Vertically opposite angles) ...(i)

Now, in ΔPOR and ΔQOS,

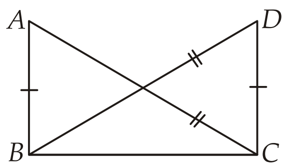
$PR = QS$ (given)

$\angle P = \angle Q$ (given)

$\angle POR = \angle QOS$ [from (i)]

$\therefore \quad \Delta POR = \Delta QOS$ (By ASA congruence rule)

(v)



In ΔABC and ΔDCB,

$AB = DC$ (given)

$AC = DB$ (given)

BC is common.

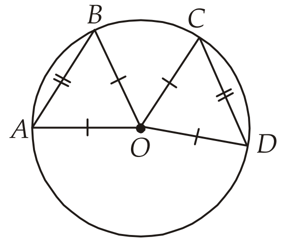
$\Delta ABC \cong \Delta DCB$

(By SSS congruence rule)

HOTS QUESTIONS

1. Given that : A circle with centre O.

$OA = OB = OC = OD$ radii and chord $AB = CD$.



In ΔOAB and ΔOCD,

$OB = OD$ (given)

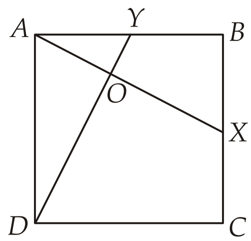
$OA = OC$ (given)

$AB = CD$ (given)

$\Delta OAB \cong \Delta OCD$ (By SSS congruence rule)

2. Given that : $ABCD$ is a square.

To prove : If $AX = DY$, then AX and DY are at right anlges.



Proof: In ΔADY and ΔBAX,

$\angle A = \angle B = 90°$

$AD = BA$ (given)

Hypotenuse DY = hypotenuse AX (given)

So, $\Delta ADY \cong \Delta BAX$ (By RHS congruence rule)

$\Rightarrow \quad AY = BX$ (CPCT)

$\Rightarrow \quad \angle AXB = \angle DYA$ (CPCT)

Let the point of intersection of AX and DY be O.

Now, in ΔDAY and ΔABX

$\angle A$ is common.

$\angle OYA = \angle AXB$

So, $\angle AOY$ must be equal to $\angle ABX$ which is 90°

$\therefore \quad \angle AOY = 90°$

Now, $\angle AOY + \angle XOY = 180°$

(AX is a straight line segment)

$\Rightarrow \quad \angle XOY = 180° - 90° = 90°$

Thus, if $AX = DY$, then AX and DY are at right angles.